

1. $r = f(\theta)$

$$\text{slope of tangent} = \frac{dy}{dx} = \frac{d(r \sin \theta)}{d(r \cos \theta)} = \frac{r \cos \theta + \sin \theta \frac{dr}{d\theta}}{-r \sin \theta + \cos \theta \frac{dr}{d\theta}}$$

Answer : C

2. $\frac{x^2}{2} - \frac{y^2}{5} = 1$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$$a = \pm\sqrt{2} \quad b = \pm\sqrt{5}$$

$$c^2 = a^2 + b^2 = 2 + 5 = 7$$

$$c = \pm\sqrt{7}$$

$$\text{Vertices} = (\pm a, 0) = (\sqrt{2}, 0), (-\sqrt{2}, 0)$$

$$\text{Foci} = (\pm c, 0) = (\sqrt{7}, 0), (-\sqrt{7}, 0)$$

$$\text{Asymptotes} = \pm \frac{b}{a} x = \pm \frac{\sqrt{5}}{\sqrt{2}} x$$

$$= \pm \frac{\sqrt{10}}{2} x$$

Correct graph. B

3. $\int 5 e^x \sec(e^x + 3) dx$

$$\text{Let } u = e^x + 3 \quad du = e^x dx$$

$$= 5 \int \sec(u) du = 5 \ln |\tan(u) + \sec(u)| + C$$

$$= 5 \ln |\tan(e^x + 3) + \sec(e^x + 3)| + C$$

4 (a) True. The rotation of the function about x-axis creates cylindrical shape parallel to the axis

(b) False. Disk/washers may be used by integration w.r.t y

(c) True. If disk/washers are perpendicular to the axis of revolution x must be used and if shells are parallel

$$5. \sum_{k=1}^{\infty} \frac{x^k}{7^k} \quad a_k = \frac{x^k}{7^k}$$

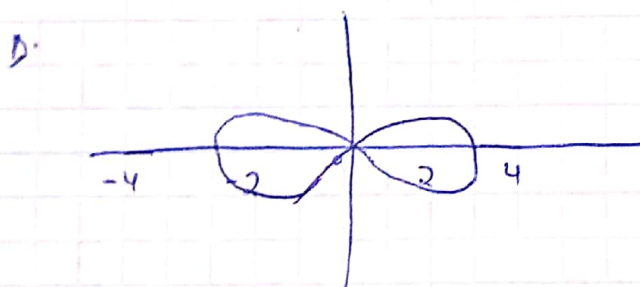
$$r = \lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = \lim_{k \rightarrow \infty} \frac{x^{k+1}}{7^{k+1}} \cdot \frac{7^k}{x^k} = \frac{x}{7}$$

$$\sum_{k=1}^{\infty} \frac{x^k}{7^k} \text{ converges for } \frac{x}{7}$$

$$\text{When } x = 7 \quad \sum_{k=1}^{\infty} \frac{x^k}{7^k} \text{ diverges.}$$

Series converges for $x < 7$

$$6. \quad r^2 = 9 \cos 2\theta$$



$$7. \sum_{k=1}^{\infty} \frac{k \cdot 7^{k-1}}{(k+1)!}$$

$$\text{Series } e^x \text{ about } 0 = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$\frac{e^x - 1}{x} = \frac{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots\right) - 1}{x}$$

$$= 1 + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$f(x) = \sum_{k=1}^{\infty} \frac{x^k}{(k+1)!}$$

$$f'(x) = \frac{e^x (x-1) + 1}{x^2} = \sum_{k=1}^{\infty} \frac{k x^{k-1}}{(k+1)!}$$

$$f'(7) = \sum_{k=1}^{\infty} \frac{k \cdot 7^{k-1}}{(k+1)!} = \frac{e^{7+1}}{49}$$

$$8. \quad \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

$$\begin{aligned} f(11x) &= \frac{1}{1-11x} = (1-11x)^{-1} \\ &= 1 - 11x + (11x)^2 + (11x)^3 + \dots \\ &= \sum_{k=0}^{\infty} (11x)^k \quad B \end{aligned}$$

$$a_k = 11^k$$

$$a_{k+1} = 11^{k+1}$$

$$\text{radius} = \lim_{k \rightarrow \infty} \frac{a_k}{a_{k+1}} = \lim_{k \rightarrow \infty} \frac{11^k}{11^{k+1}} = \frac{1}{11}$$

$$\text{Interval} = |x| \leq R$$

$$= |x| \leq \frac{1}{11}$$

$$= \left(-\frac{1}{11}, \frac{1}{11}\right)$$

$$9. (a) \quad f(x) = \tan^{-1}(4x) = (4x) - \frac{(4x)^3}{3} + \frac{(4x)^5}{5}$$

$$1^{\text{st}} \text{ term} = 4x$$

$$2^{\text{nd}} \text{ term} = -\frac{64}{3} x^3$$

$$3^{\text{rd}} \text{ term} = \frac{1024}{5} x^5$$

$$(b) \quad \sum_{k=0}^{\infty} \frac{(-1)^k 4^{2k+1}}{2k+1} \cdot x^{2k+1}$$

$$(c) \quad |(4x)^2| \leq 1$$

$$|4x| \leq 1 \quad |x| \leq \frac{1}{4} \quad \left[-\frac{1}{4}, \frac{1}{4}\right]$$

$$10 \quad n = 2, \quad a = 0$$

$$f(x) = f(0) + f'(0)(n-0) + f''(0)(n-0)$$

$$P(0) = f(0)$$

$$P'(0) = f'(0)$$

$$P''(0) = f''(0)$$

$$P(0) = f(0), P'(0) = f'(0), P''(0) = f''(0) \quad C$$

$$11 \quad \sum_{k=1}^{\infty} \frac{1}{k+3}$$

$$S_1 \quad k=1 \quad \text{sum} = \frac{1}{1+3} = \frac{1}{4}$$

$$S_2 \quad k=2 \quad \frac{1}{1+3} + \frac{1}{2+3} = \frac{9}{20}$$

$$S_3 \quad k=1, 2, 3 \quad \frac{9}{20} + \frac{1}{3+3} = \frac{37}{60}$$

$$S_4 \quad k=1, 2, 3, 4 \quad \frac{37}{60} + \frac{1}{4+3} = \frac{319}{420}$$

$$12. \quad \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right)$$

$$S_n = \frac{1}{1} - \frac{1}{n+1} = 1 - \frac{1}{n+1}$$

$$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+1} \right) = 1 - 0 = 1$$

$$13 \quad \left(4, \frac{2\pi}{3} \right) \quad r = 4$$

$$\theta = \frac{2\pi}{3}$$

$$x = r \cos \theta = 4 \cos \left(\frac{2\pi}{3} \right) = -2$$

$$y = r \sin \theta = 4 \sin \left(\frac{2\pi}{3} \right) = 2\sqrt{3}$$

$$= (-2, 2\sqrt{3})$$

$$14 \quad y^2 = -25x$$

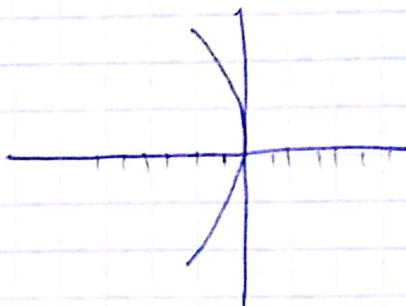
$$\text{focus} \quad 4p = -25$$

$$p = -\frac{25}{4}$$

$$\text{focus} = \left(-\frac{25}{4}, 0 \right)$$

$$\text{directrix}, x = -\frac{25}{4}$$

Graph C



15.

$$a_n = \{0.6\}^n$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} (-1)^n (0.6)^n = 0$$

Monotonically converged at 0.

B